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Properties of Dom-Set Chromatic Number of Graphs

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ABSTRACT: Let G be a graph. For a non-trivial connected graph G , let $C: V(G) \rightarrow \mathbb{N}$ be a vertex coloring of G where adjacent vertices may be assigned the same color. For a dom-coloring set $S \subseteq (G)$, define $C(S)$ of colors assigned to the vertices of S by $C(S) = \{C(v): v \in S\}$. For a vertex v in a graph G , let $N(v)$ be the neighbourhood of v (the dom-coloring set of all vertices adjacent to v in G). The neighbourhood color set $NC(v) = C(N(v))$ is the set of colors of the neighbours of v . The new coloring c is called dom-set coloring if $NC(u) \neq NC(v)$ for every pair u, v of adjacent vertices of G . The minimum number of colors required of such a coloring is called the dom-set chromatic number of G and denoted by $\gamma_{(d-c)}(G)$. In this paper, we explain some properties of dom-set chromatic number of graphs.

KEYWORDS: Dom- set chromatic number, tree and complete graph.

I. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definition of graph theory as found in [1]. Let $G = (V, E)$ be a graph. The open neighborhood $N(v)$ of a vertex $v \in V(G)$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. An induced subgraph $G[S]$, where S of a graph G is a graph formed from a subset S of the vertices of G and all of the edges connecting pairs of vertices in S . A graph in which every pair of vertices is joined by exactly one edge is called complete graph. A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both end points in the same subset, and each vertex of V_1 is connected to every vertex of V_2 and vice versa. A star graph S_n is the complete bipartite graph $K_{1,n-1}$ (A tree with one internal node and $n-1$ leaves).

The path and cycle of order n are denoted by P_n and C_n respectively. For any two graphs G and H , we define the cartesian product, denoted by $G \times H$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) iff either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$.

A subset S of V is called a dominating set if every vertex in $V-S$ is adjacent to at least one vertex in S . The dominating set is minimal dominating set if no proper subset of S is a dominating set of G . The domination number γ is the minimum cardinality taken over all minimal dominating set of G . A γ -set is any minimal dominating set with cardinality γ .

A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The minimum number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A dominator coloring of G is a proper coloring of G in which every vertex of G dominates at least one color class. The dominator chromatic number is denoted by $\chi_d(G)$ and is defined by the minimum number of colors needed in a dominator coloring of G . This concept was introduced by Raluca Michelle Gera in 2006[2].

In a proper coloring C of G , a color class of C is a set consisting of all those vertices assigned the same color. Let C^l be a minimal dominator coloring of G . We say that a color class $c_i \in C^l$ is called a non-dominated color class (n -color class) if it is not dominated by any vertex of G . These color classes are also called repeated color classes. For a non-trivial connected graph G , let $C: V(G) \rightarrow \mathbb{N}$ be a vertex coloring of G where adjacent vertices may be assigned the same color. For a dom-coloring set $S \subseteq (G)$, define $C(S)$ of colors assigned to the vertices of S by $C(S) = \{C(v): v \in S\}$. For a vertex v in a graph G , let $N(v)$ be the neighbourhood of v (the dom-coloring set of all vertices adjacent to v in G). The neighbourhood color set $NC(v) = C(N(v))$ is the set of colors of the neighbours of v . The new coloring c is called dom-set coloring if $NC(u) \neq NC(v)$ for every pair u, v of adjacent vertices of G . The minimum number of colors



required of such a coloring is called the dom-set chromatic number of G and denoted by $\gamma_{(d-c)_S}(G)$. For a graph G with dom-chromatic number k , let c be a proper k -dom-coloring of G . Suppose that u and v are adjacent vertices of G . Since $(u) \in NC(v)$ and $c(u) \notin NC(u)$ it follows that $NC(u) \neq NC(v)$. Hence every proper k -dom-coloring of G is also called set k -dom-coloring of G . Therefore, for every graph G , $\gamma_{(d-c)_S}(G) \leq \gamma_{d-c}(G)$.

OBSERVATION 2.1:

If u and v are two adjacent vertices in a graph G such that $N(u) - v = N(v) - u$ then $c(u) \neq c(v)$ for every set coloring c of G . Furthermore, if $S = N(u) - v = N(v) - u$ then $c(u), c(v) \notin c(S)$.

OBSERVATION 2.2:

If G is a non empty bipartite graph, then $\gamma_{(d-c)_S} = 2$.

THEOREM 1:

If G is a connected graph with $\gamma_{d-c}(G) \geq 3$, then $\gamma_{(d-c)_S}(G) \geq 3$. PROOF:

Assume, to the contrary, that there exists a connected graph G with $\gamma_{d-c}(G) \geq 3$ for which there exists a set 2-coloring $c: V(G) \rightarrow \{1,2\}$. Since $\chi(G) \geq 3$, it follows that G contains an odd cycle

$C: v_1, v_2, \dots, v_l, v_1$ where $l \geq 3$ is an odd integer. Consider the (cyclic) dom-color sequence $S: (v_1), c(v_2), \dots, c(v_l), c(v_1)$. By a block of S , we mean a maximal subsequence of S consisting of terms of the same color.

Claim that S cannot contain a block with an even number of terms.

Without loss of generality, $c(v_l) = 2, c(v_i) = 1$ for $1 \leq i \leq a$ where a is an even integer with $2 \leq a \leq l - 1$ and $c(v_a) = 2$. Thus $NC(v_i) \in \{\{1\}, \{1,2\}\}$ for $1 \leq i \leq a$. Since $NC(v_1) = \{1,2\}$ and

c is set coloring it follows, $NC(v_i) = \begin{cases} \{1\} & \text{if } i \text{ is even} \\ \{1,2\} & \text{if } i \text{ is odd} \end{cases}$

for $1 \leq i \leq a$. This implies that $N(v_a) = \{1\}$, which is impossible since $c(v_{a+1}) = 2$. Hence either

- i. $c(v_i) = 1$ for all $i (1 \leq i \leq l)$
- ii. S contains even number of blocks each of which has an odd number of terms.

If (i) occurs, then $NC(v_i) \in \{\{1\}, \{1,2\}\}$ for $1 \leq i \leq l$. Since l is odd, there is an integer $(1 \leq j \leq l)$ such that $NC(v_j) = NC(v_{j+1})$ which is impossible.

If (ii) occurs, then l is even which is impossible.

Hence our assumption is wrong. Therefore, every connected graph with $\gamma_{d-c}(G) \geq 3$ must have

$$\gamma_{(d-c)_S}(G) \geq 3$$

COROLLARY 1:

A non-empty graph G has dom-set chromatic number 2 iff G is bipartite.

PROOF:

Assume $\gamma_{(d-c)_S}(G) = 2$

To prove G is bipartite.

Since $\gamma_{(d-c)_S}(G) = 2$. Then G has a dom-set coloring with colors $\{1,2\}$. This coloring is automatically proper because if u and v are adjacent and $c(u) = c(v)$, then $c(u)$ appears in both $N(u)$ and $N(v)$ and with only 2 colors it is impossible to make $c(N(u)) \neq c(N(v))$. Thus $\chi(G) \leq 2$.

Therefore, G is bipartite.

Conversely,

Assume G is bipartite.

To prove $\gamma_{(d-c)_S}(G) = 2$.

The proof is obvious by observation 2.2.

COROLLARY 2:

If G is a 3-dom-chromatic graph, then $\gamma_{(d-c)_S}(G) = 3$

PROOF:

Clearly $\gamma_{(d-c)_S}(G) \geq \gamma_{d-c}(G)$ (any dom-set coloring is a proper coloring). Since $\gamma_{d-c}(G) = 3$, we have

$$\gamma_{(d-c)_S}(G) \geq 3 \rightarrow (1)$$



No additional color is forced beyond the ordinary dom-chromatic number when $\gamma_{d-c}(G) = 3$.

$${}_{(d-c)\mathcal{S}}(G) \leq 3 \rightarrow (2)$$

From (1) and (2), ${}_{(d-c)\mathcal{S}}(G) = 3$.

COROLLARY 3:

For each integer $n \geq 3$, ${}_{(d-c)\mathcal{S}}(C_n) = \gamma_{d-c}(C_n)$. PROOF:

Case i) n is even ($n \geq 4$)

Then C_n is bipartite, which implies $\gamma_{d-c}(C_n) = 2$. By Corollary 2.1, ${}_{(d-c)\mathcal{S}}(C_n) = 2$ iff C_n is bipartite.

Therefore, ${}_{(d-c)\mathcal{S}}(C_n) = \gamma_{d-c}(C_n)$.

Case ii) n is odd ($n \geq 3$)

Let C_n be non-bipartite, which implies $\gamma_{d-c}(C_n) = 3$. By Corollary 2.2, If G is 3 dom-chromatic then ${}_{(d-c)\mathcal{S}}(G) = 3$. This implies, ${}_{(d-c)\mathcal{S}}(C_n) = 3$.

Therefore, ${}_{(d-c)\mathcal{S}}(C_n) = \gamma_{d-c}(C_n)$.

THEOREM 2:

For $n \geq 2$ and $0 \leq t \leq n$, ${}_{(d-c)\mathcal{S}}(G_{n,t}) = n$. PROOF:

Let G_n , denote the graph of order $n + 1$ obtained from K_n with $V(K_n) = \{v_1, v_2, \dots, v_n\}$ by adding t new vertices u_1, u_2, \dots, u_t and joining each u_i to v_i for $1 \leq i \leq t$. The result follows immediately if $n = 2$ or $t = 0$, so we assume that $n \geq 3$ and $1 \leq t \leq n$.

Since $\gamma_{d-c}(G_{n,t}) = n$, we have $\gamma_{(d-c)\mathcal{S}}(G_{n,t}) \leq n$. Suppose that ${}_{(d-c)\mathcal{S}}(G_{n,t}) = k \leq n - 1$ and let there be given a set k -dom-coloring of $G_{n,t}$ using the colors in $\mathbb{N}k$. Permuting colors if necessary, we can obtain a set k -dom-coloring $c: (G_{n,t}) \rightarrow \mathbb{N}k$ such that $c(V(K_n)) = \mathbb{N}l$ for some $l \leq k$.

Let X be the subset of $V(K_n)$ such that for every $x \in X$ there exists a vertex $y \in X - x$ for which $c(y) = c(x)$. Since c uses at most $n - 1$ colors, $|X| \geq 2$ and furthermore, since each of the vertices in $V(K_n) - X$ receives a unique color $n - |X| + 1 \leq l$.

For each, $x \in X$, either

- i. $NC(x) = \mathbb{N}l$
- ii. $NC(x) = \mathbb{N}l \cup c(u)$ if u is the end vertex adjacent to x and $c(u) \notin \mathbb{N}l$.

Since at most one of the $|X|$ vertices can have the neighbourhood color set $\mathbb{N}l$, at least $|X|-1$ colors not in $\mathbb{N}l$ are needed to color the end vertices so that the vertices in X have distinct neighbourhood color set, that is $k \geq l + |X| - l \geq n$

It is a contradiction. Therefore, ${}_{(d-c)\mathcal{S}}(G_{n,t}) = n$

THEOREM 3:

For every complete k -partite graph G , ${}_{(d-c)\mathcal{S}}(G) = k$. PROOF:

Given that ${}_{(d-c)\mathcal{S}}(G) = k$. We know that ${}_{(d-c)\mathcal{S}}(G) \leq \gamma_{d-c}(G) = k$. Therefore, ${}_{(d-c)\mathcal{S}}(G) \leq k$.

Assume that the statement is false. Let ${}_{(d-c)\mathcal{S}}(G) \leq k - 1$, $k \geq 4$. Suppose that the partite

sets of G are V_1, V_2, \dots, V_k . Let there be given a set $(k - 1)$ coloring $c: V(G) \rightarrow \mathbb{N}k-1$ of G .

We claim that for each partite set $V_i (1 \leq i \leq k)$ the coloring $c_i = c \setminus V(G) - V_i$ is the set dom-coloring of $G - V_i$, which is complete $(k - 1)$ partite graph. Let u and v be adjacent vertices in

$G - V_i$. In G we have $NC_c(u) \neq NC_c(v)$. Since $NC_c(u) = NC_{c_i}(u) \cup c(V_i)$ and

$NC_c(v) = NC_{c_i}(v) \cup c(V_i)$, it follows $NC_{c_i}(u) \neq NC_{c_i}(v)$. This implies that the coloring c_i of

$G - V_i$ is set coloring.

Since $\chi(G - V_i) = k - 1$, it follows that $c(V(G) - V_i) = \mathbb{N}k-1$.

Thus $NC_c(x) = \mathbb{N}k-1$ for every vertex x of V_i . Since the partite set V_i was chosen arbitrarily

$NC_c(x) = \mathbb{N}k-1$ for every vertex x of G , which is impossible.

Therefore, ${}_{(d-c)\mathcal{S}}(G) = k$.



COROLLARY 4:

For each pair k, n of integers with $2 \leq k \leq n$, there is a connected graph G of order n with $\gamma_{(d-c)_S}(G) = k$.

OBSERVATION 3.3:

Let G be a graph of order $n \geq 2$. Then $\gamma_{d-c}(G) = n - 1$ if and only if $\omega(G) = n - 1$.

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